① Q No → Find the radical axis and the length of the common chord of the two circles:
$x^2+y^2+3x+5y+4=0$ and $x^2+y^2+5x+3y+4=0$

Ans. → given circles are

$S_1 \equiv x^2+y^2+3x+5y+4=0$

and $S_2 \equiv x^2+y^2+5x+3y+4=0$

the eqn. of radical axis is given by

$S_1 - S_2 = 0$

$3x-5x+5y-3y=0$

$-2x+2y=0$

$x-y=0$

$ax+by-bx-ay=0$

$x(a-b)+y(b-a)=0$

or, $x(a-b)-y(a-b)=0$

or, $(a-b)(x-y)=0$

$\therefore x-y=0$ required eqn. of radical axis

The centre C of the circle ① $x^2+y^2+3x+5y+4=0$

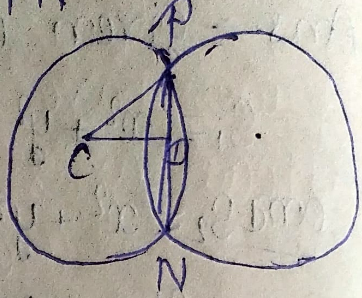
is $(-g,-f)=\left(-\frac{3}{2},-\frac{5}{2}\right)$ and $c=4$

and its radius is $\sqrt{g^2+f^2-c}=\sqrt{\left(-\frac{3}{2}\right)^2+\left(\frac{5}{2}\right)^2-4}$

$=\sqrt{\frac{9}{4}+\frac{25}{4}-4}=\sqrt{\frac{9+25}{4}-4}=\sqrt{\frac{34}{4}-4}$

$= \sqrt{\frac{34-16}{4}} = \sqrt{\frac{18}{4}} = \frac{\sqrt{9}}{2} = \frac{3}{\sqrt{2}}$

Now draw $CD \perp PN$ from $C$ to $PN$



Then $CD = \frac{-\frac{3}{2} - \left(-\frac{5}{2}\right)}{\sqrt{1^2 + (-1)^2}}$

$= \frac{\frac{-3+5}{2}}{\sqrt{1+1}} = \frac{\frac{2}{2}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

Radius of $S_1$, $PN = \frac{3}{\sqrt{2}}$

in right angled triangle $PCD$

$\therefore PC^2 = CD^2 + PD^2$

$\therefore AD^2 = AC^2 - CD^2 = \left(\frac{3}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{9}{2} - \frac{1}{2} = \frac{9-1}{2} = \frac{8}{2} = 4$

$\therefore AD = 2$

Thus the required length of the common chord

$= AB = 2 \times AD = 2 \times 2 = 4$ Ans

② QN.→ Find the radical axis of the circles $x^2+y^2=2x$ and $2x^2+2y^2-3y=5$.

Ans.→ Equations of the given circles are

$x^2 + y^2 - 2x = 0$ — ①

and $2x^2+2y^2-3y-5=0$ — (2)

multiplying (1) by 2, we have,

$2x^2+2y^2-4x=0$ — (3)

Hence, the radical axis of (1) and (2) and (2) and (3) is given by

$$+4x-3y-5=0$$

$$\text{or, } 4x-3y-5=0$$

(3) Qn.→ Find the Co-ordinates of the radical Centre of the three circles $x^2+y^2+4x+7=0$, $2x^2+2y^2+3x+5y+9=0$ and $x^2+y^2+y=0$ ~~Hence find the equation of the circle which cut these circles orthogonally.~~

Ans.→ Equation of the given circles are

$x^2+y^2+4x+7=0$ — (1)

$2x^2+2y^2+3x+5y+9=0$ — (2)

$x^2+y^2+y=0$ — (3)

multiplying (3) by (2), we have

$2x^2+2y^2+2y=0$

Radical axis of (1) and (2)

$4x-y+7=0$ — (a)

Radical axis of (2) and (3)

$3x+2y+9=0$ — (b)

the point of intersection of (a) and (b) with give the radical centre

$$4x-y+7=0$$

$3x+3y+9=0$

or, $x+y+3=0$ —ⓑ

Solving the point of intersection of ⓐ and ⓑ will give the radical center

$$4x-y+7=0$$

$$x+y+3=0$$

$$\therefore \frac{x}{-3-7} = \frac{y}{\cancel{12}\,7-12} = \frac{1}{4+1}$$

$$\frac{x}{-10} = \frac{y}{-5} = \frac{1}{5}$$

$x=-2$

$y=-1$

Hence the required radical centre is $(-2,-1)$

④ Q.No.→ Find the equation of the circle whose diameter is the common chord of the circles $x^2+y^2+2x+3y+1=0$ and $x^2+y^2+4x+3y+2=0$

Ans.→ Equation of the given circles are

$$x^2+y^2+2x+3y+1=0$$

and $x^2+y^2+4x+3y+2=0$

Therefore the equation of the common chord is.

$$x^2+y^2+2x+3y+1 - x^2+y^2+4x+3y+2=0$$

or, $2x+1=0$ —①

∴ Equation of the circle through the extremities of the common chord is

$x^2+y^2+2x+3y+1+K(2x+1)=0$

or, $x^2+y^2+2x(K+1)+3y+K+1=0$ — ②

Centre of this circle ② is $\left(-K-1, -\frac{3}{2}\right)$

If the common chord ① is a diameter, then the centre lies on ①

∴ $-2(K+1)+1=0$, or, $K=-\frac{1}{2}$

Substituting this value of K in eqn. ②, the required equation of the circle is

$x^2+y^2+2x\left(-\frac{1}{2}+1\right)+3y-\frac{1}{2}+1=0$

or, $x^2+y^2+x+3y+\frac{1}{2}=0$

⑤ QN.→ Find the limiting points of the system of circles of which two members are

$x^2+y^2-6x-6y+4=0$ and $x^2+y^2-2x-4y+3=0$.

Ans.→ ∴ the eqn. of two circles is

$x^2+y^2-6x-6y+4=0$ — ①

and, $x^2+y^2-2x-4y+3=0$ — ②

Hence, the radical axis of ① and ② is given by

$-4x-2y+1=0$

or, $4x+2y-1=0$

Hence, the eqn. of co-axial circle with the given circle can be written as

$x^2+y^2-2x-4y+3+K(4x+2y-1)=0$

$x^2+y^2-2x(1-2K)-2y(2-K)+3-K=0$

Evidently the centre of the

~~radius~~ $= \cancel{\sqrt{g^2+f^2-c}}$

circle $=(1-2K),(2-K)$

radius $=\sqrt{g^2+f^2-c}$

radius of the circle is

$=\sqrt{(1-2K)^2+(2-K)^2-(3-K)}$

for limiting points, the radius $=0$

$(1-2K)^2+(2-K)^2-(3-K)=0$

$1-4K+4K^2+4-4K+K^2-3+K=0$

$5K^2-7K+2=0$

$5K^2-5K-2K+2=0$

$5K(K-1)2(K-1)=0$

$(K-1)(5K-2)=0$

$K-1=0 \qquad 5K-2=0$

$K=1 \qquad 5K=2$

$\therefore K=1 \qquad \therefore K=\frac{2}{5}$

Centre of the circle

$=(1-2K),(2-K)$

putting $K=1$, and $K=\frac{2}{5}$

$(-1,1)\ (\frac{1}{5},\frac{8}{5})$

Hence, the limiting points are

$(-1,1)$ and $(\frac{1}{5},\frac{8}{5})$